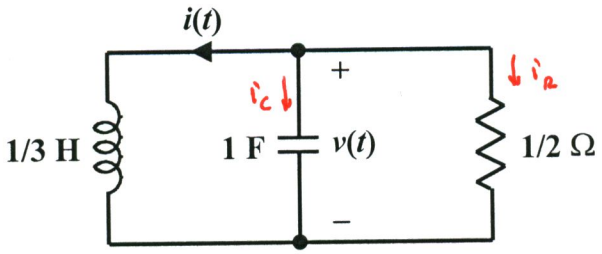
EE 2240

Problem #08

Find $i(t)$ for $t \geq 0$ if $v(0) = -1/3\,\text{V}$ and $i(0) = 1\,\text{A}$.



$$v = \frac{1}{3}\frac{di}{dt} \Rightarrow \left.\frac{di}{dt}\right|_{t=0} = 3v(0) = -1\,\text{V}$$

$$i_C = 1\,\frac{dv}{dt} = \frac{1}{3}\frac{d^2i}{dt^2}, \qquad i_R = 2v = \frac{2}{3}\frac{di}{dt}$$

$$\frac{1}{3}\frac{d^2i}{dt^2} + \frac{2}{3}\frac{di}{dt} + i = 0 \Rightarrow \frac{d^2i}{dt^2} + 2\frac{di}{dt} + 3i = 0$$

$$r^2 + 2r + 3 = (r+1)^2 + 2 \Rightarrow r = -1 \pm j\sqrt{2}$$

$$\therefore\ i(t) = e^{-t}\left(K_1 \cos\sqrt{2}t + K_2 \sin\sqrt{2}t\right)$$

$$\frac{di}{dt} = -e^{-t}\left(K_1 \cos\sqrt{2}t + K_2 \sin\sqrt{2}t\right) + e^{-t}\left(-\sqrt{2}K_1 \sin\sqrt{2}t + \sqrt{2}K_2 \cos\sqrt{2}t\right)$$

$$\left.\begin{aligned} i(0) &= K_1 = 1 \\ \left.\frac{di}{dt}\right|_{t=0} &= -K_1 + \sqrt{2}K_2 = -1 \end{aligned}\right\} \quad \begin{aligned} K_1 &= 1 \\ K_2 &= 0 \end{aligned}$$

$$i(t) = e^{-t}\cos\sqrt{2}t \quad \text{A}, \quad t \geq 0$$